NUMERACY LESSON TO SUPPORT NEWTON'S CRADLE ADST PROJECT

BALANCED EQUATIONS

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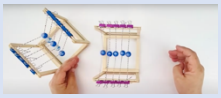
"Education is all a matter of building bridges." ~ Ralph Ellison

This unit is designed to supplement the Newton's Cradle section of the 9 Applied Design, Skills and Technology (ADST) Lessons found at https://digmore.prn.bc.ca/helpful-resources/adst/. *ADST support exists for SD60 teachers, including materials, tools and training. Please contact your administrator if you'd like to access this support.*

LESSON TITLE	LITERACY/ NUMERACY TASKS	BRIEF OVERVIEW OF LESSON
Balanced Equations Supports "Newton's Cradle" ADST Lesson Grade 6 	**Numeracy Task: Keeping equations balanced** (preservation of equality) https://www.youtube.com/watch?v=KTMXCagRTMU Curricular Competency: Explain and justify mathematical ideas and decisions Communicate mathematical thinking in many ways Represent mathematical ideas in concrete, pictorial, and symbolic forms Content: one-step equations with whole-number coefficients and solutions	After creating Newton's Cradle, students will explore the concept of balance as applied to equations (the preservation of equality).

TARGET AUDIENCE: Grade 6 students (math - balanced equations) though the game can be played by younger grades.
This lesson is adapted from "Well Played: Building Mathematical Thinking Through Games and Puzzles" (Dacey, Gartland & Lynch, 2015

PREP WORK:

- Photocopy "Preservation of Equality" worksheets on regular paper @ 1 set per person.
- Photocopy Equal Values Recording Sheet @ 1 per pair, double-sided
- Photocopy Equal Values Game on cardstock @ 1 per foursome. Laminate.
- Watch this YouTube if you need a refresher on how to teach/explain Balanced Equations, c/o Daniel Edey. The Preservation of Equality worksheet you have (below) is based on his YouTube but isn't an exact copy, so you can lean on his explanations if you need to.

MATERIALS LIST:

- Preservation of Equality worksheet @ 1 set per person
- Equal Values Recording Sheet @ 1 per pair, double-sided

- Equal Values Game @ 1 per foursome.
- Pencils
- Calculator (optional)
- Game Direction Sheet projected onto white-board or printed out (optional)
- Counters – about 30 per student

QUESTIONS: Can students explain and demonstrate "preservation of equality" and "balanced equations"?

GRADE 6 CURRICULUM:

- **Gr 5 & 6 Big Ideas:** Computational fluency and flexibility with numbers extend to operations with larger (multi-digit) numbers.
- **Gr 5 & 6 Curricular Competencies:**
  - Develop mental math strategies and abilities to make sense of quantities
  - Model mathematics in contextualized experiences
- **Gr 6 Content:** one-step equations with whole-number coefficients and solutions

VOCABULARY:

Equal: = means *the **same***
Balance: when both sides have the **same** quantity or mass
Equation: An equation is simply a statement in math in which two things on **equal**. There are two expressions, one on each side of an equal sign.
Preservation of Equality: Preservation of equality is **when each side of the equation is changed in the same way**. In other words "What you do to one side, you must do to the other."
Balanced Equation: A balanced equation is **an equation where both sides are equal to the same amount**.

INTRODUCE THE LESSON:

Newton's Cradle is connected to balance and equality. If you pull up three balls and let go, three balls will swing out from the other side. If you pull up one ball, only a single ball will swing on the other side. We could have a Newton's Cradle as big as a bus and pull up 75 balls; guess how many would swing on the other side once the energy was transferred? You bet. 75. The cradle remains in a state of balance. Today we are going to take the idea of balance and think about how that applies to math when we balance equations.

TEACH THE LESSON:

1. Distribute 30 counters to each student.
2. Distribute the worksheet to each student. Have the students perform the activity and fill in the blanks. Refer to Daniel Edey's video: Grade 6 Math Lesson – keeping equations balanced for a "flipped classroom" example of how to teach this lesson.
   a. You are welcome to follow up the worksheet (below) with any textbook work on Balanced Equations / Preservation of Equality that you see fit.
3. ABOUT THE GAME. To help learners better understand the idea of *balancing equations & preservation of equality*, play the "Equal Values" game (pp. 77-80) from "Well

Played: *Building Mathematical Thinking Through Number Games and Puzzles Grades 3-5,* (Dacey, Gartland & Lynch, 2015)." An adapted version of *Dacey et al's* game follows:

a. "This game requires students to identify two multiplication expressions that have the same value. The expressions in the game emphasize the properties of multiplication. The distributive property allows students to build on facts they know, for example, thinking of 4 x 8 as the same as 2 x 8 + 2 x 8. Learners might also find 8 x 7 by thinking about 10 x 7 – 2 x 7. The commutative property is also helpful, as students may be able to find 9 x 3 by thinking of 3 x 9. Later, to generalize the properties, students must also apply them to numbers greater than nine. [The game] also [helps students practice] mental computation to find products of single-digit numbers and multiples of tens and hundreds; for example, it is helpful to multiply 5 x 2 first to find 45 x 5 x 2.
b. After finding two equal expressions, students are asked to record the expression on the Equal Values Recording Sheet (below) on either side of an equal sign. Many students misconstrue the equal sign to mean "write your answer here" (Ginsberg and Ertle 2008) and this game helps them see that the equal sign is meant to show the balance between equations. Additionally, some students may also incorrectly assume that only one number follows an equal sign, when in fact any expression with the same value can be written there.

EQUAL VALUES GAME DIRECTIONS (Adapted from pp. 78-79 of "Well Played*"). 4 students play; Team A vs Team B*

GOAL: Get more pairs of cards (A.K.A. "A Pack") that have equal values

- Give each team a card with the equal sign
- Shuffle the cards. Deal each team four cards faceup for all to see. Put the other cards face-down in a pile.
- On each turn, you can do one of three things:
  1. Find two of your cards that have an equal value. Set this pack beside you. Replace them with two cards from the top of the file.
  2. Trade one of your cards with one of the other team's cards when you are able to make a pack Set this pack beside you. Replace your card with a card from the top of the pile.
  3. Draw a card from the top of the pile and add it to your cards.
- When a pack is made, both teams must agree on the product and then one player records their thinking.
- If no cards are left in the deck, you can still have a turn, but you don't take a card.
- The game ends when no team can make another pack.
- The team with more packs wins.

4. INTRODUCE THE GAME

On the board, the teacher writes the following list of expressions and asks students to find two expressions that have the same value.

4 x 37
30 + 4 + 7 x 4
2 x 37 + 2 x 37
37 X 4

In pairs, students work together. Bob and Sue identify the first and last expressions as having the same value and explain their thinking: *You can change the order of the numbers (aka factors).*

Min and Harshmeet note that the first two expressions are equal because you can split the factors.

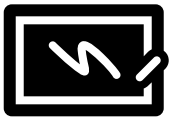
Pete and Fa say, "Wait, if the first one is the **same** as the second one and the last one, then all three must be **equal**."

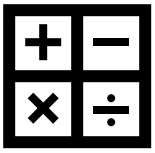
Bob is entirely too excited by this idea and suggests that all 4 expressions have the same value. The teacher divides the class into 4 sections and assigns each section one of the above 4 expressions to compute. Is Bob correct or incorrect?

Once all products are found to equal 148, the teacher applauds their use of math vocabulary (same, equal) shows them how to **PLAY THE EQUAL VALUES GAME.** She projects and reads the rules (above). She gets four students (Pair A and Pair B) to model the game-play as per the rules above. Calculators may be used with students who have emergent multiplication skills.

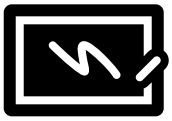
5. PRO TIPS
    a. Recording expressions on the Recording Sheet (below) slows down the game but reinforces thinking and understanding.
    b. Colour the equal sign cards so they don't get lost in the deck.
    c. Encourage the students who are unsure whether two expressions are equal or not to replace the larger numbers with smaller ones and see what they think. This approach is an example of simplifying a problem; an important problem-solving strategy.
    d. Using a **calculator** to double-check products helps students **focus on balancing equations** instead of multiplication.
6. WHAT TO LOOK FOR WHEN STUDENTS ARE PLAYING THE GAME
    a. What strategies do students use to find equal products?
    b. Do students recognize examples of the commutative, associative or distributive properties? ← Click the link for a student-friendly definition of each property c/o mathisfun.com
    c. What language do students use when deciding whether two expressions are equal or not?
    d. Do players consider what cards their opponents have when they decide what card to give away?
7. VARIATIONS
    a. Students can make their expressions and add them to the deck once they're familiar with the game.
    b. Play with division expressions & then with both multiplication & division expressions, allowing students to see inverse relationships such as 30 x 3 = 90 ÷ 3
    c. Students can 1 versus 1 instead of 2 versus 2.

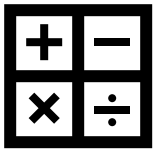
END THE LESSON:





- Exit Slip
    - Project these Exit Card Choices onto the board and have students math-journal their choice & answer:
        - How do you know that 7 x 36 = 36 x 7?
        - What are some different ways you could find 27 x 6?
        - What are two expressions you can write that are equal to 5 x 92?
- Extension
    - Post several expressions throughout the room and have a scavenger hunt. Teams can use the Equal Values Recording Sheet to record those they find with the same value.





ASSESSMENTS

A note about assessments: Your classroom observations and exit slips are more than enough fodder for assessment. The rubrics are optional and are provided for your convenience.

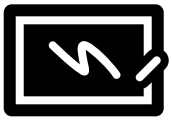
	Exploring	Developing	Proficient	Extending
I can show & tell you what preservation of equality means	I can point to the equal sign in an equation	I can tell you that both sides of an equation are equal. I can show you one or two more or less with counters.	I can communicate (by speaking and/or with counters) what preservation of equality is by demonstrating an equation we've discussed.	I can communicate in writing, speaking and with manipulatives what preservation of equality is. I can show you novel equations and preserve equality using +, -, x or ÷
I can show & tell you what a balanced equation is	I know a balance is like a scale and I can put weights on one side and make the other side balance.	I can play the game and make sets of balanced equations.	I can identify balanced equations. I can tell you why an equation is balanced.	I can identify, represent, draw and create balanced equations (using manipulatives, symbols, equations and drawings)
I can share my mathematical thinking	* Needs support to complete Worksheet * Able to participate in games with support & adaptations (calculator, developmentally appropriate questions) * Exit slip Journal incomplete or unclear	- Worksheet is mostly complete, though some parts may be inaccurate. - My journal's exit slip has a limited explanation that answers the question	- Worksheet is complete. May have some errors. - Journal's exit slip clearly communicates a logical explanation/solution	- Worksheet is complete. Few/No errors. - Journal's exit slip demonstrates flexible thinking and multiple solutions.

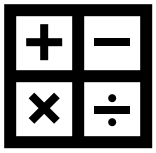
Comments: -

_____

_____

_____





PRESERVATION OF EQUALITY WORKSHEET

Name: _____ Date: _____

ACTIVITY

On your clean desk, place 3 counters on the left and 3 counters on the right. Complete the following steps:

1. Triple the number of counters on each side.
2. Remove 4 from each side.
3. Add 5 to each side.
4. Remove 2 from each side.
5. Multiply the number of counters on each side by 2.

Show your work.	

How many counters are there on the left side? _____

How many counters are there on the right side? _____

Why are the amounts of counters on each side the same?

_____

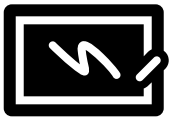
_____

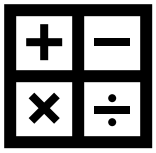
What do you think THE PRESERVATION OF EQUALITY means?

_____

_____

Be prepared to discuss your answers ☺.





Consider the following equation:

$12 = 3t$ where t is an unknown amount.
To show that equality is preserved, we can add, subtract, multiply or divide the amounts on each side.

Example: if you **add** 3 to each side,
$12 = 3t$ becomes
$12 + 3 = 3t + 3$ Preservation of equality means 3 is added to both sides.

Example: if you **subtract** 2 from each side,
$12 = 3t$ becomes
$12 - 2 = 3t - 2$ Preservation of equality means 2 is subtracted from both sides.

Example: if you multiply by 4
$12 = 3t$ becomes
$12 \times 4 = 3t \times 4$ Preservation of equality means you multiply both sides by 4.
You could also write $12 + 12 + 12 + 12 = 3t + 3t + 3t + 3t$

Example: if you divide by 3
$12 = 3t$ becomes
$\frac{12}{3} = \frac{3t}{3}$ Preservation of equality means you divide both sides by 3.

For each example below, the original equation is given with a mathematical change given. Which of the examples preserve equality?

Original	**Change**	**Preserve?**	**Why?**
$x + 4 = 6$	$x + 4 - 2 = 6 - 2$	_____	_____
$x - 5 = 7$	$x - 5 + 5 = 7 - 5$	_____	_____
$3t = 18$	$4 \times 3t = 4 \times 18$	_____	_____
$12B = 36$	$\frac{12B}{3} = \frac{36}{4}$	_____	_____

For the equation below, model the equation with counters. Use the counters to show the preservation of equality. Then draw a diagram to record your work by using symbols.

a) Addition

Adding 2

5 + 3 = 8

b) Subtraction

Subtract 9

3 x 5 = 15

c) Multiplication

Multiply by 2

12 ÷ 3 = 2 + 2

PRESERVATION OF EQUALITY ANSWER KEY

PRESERVATION OF EQUALITY ANSWER KEY

Name: _____ Date: _____

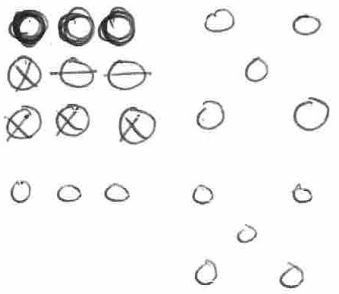
ACTIVITY

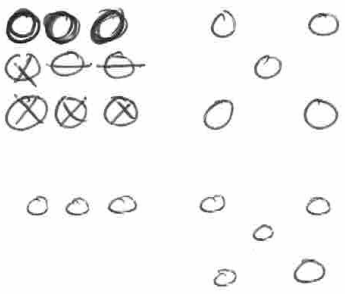
On your clean desk, place 3 counters on the left and 3 counters on the right. Complete the following steps:

1. Triple the number of counters on each side.
2. Remove 4 from each side. ×
3. Add 5 to each side. +5
4. Remove 2 from each side. −
5. Multiply the amount of counters on each side by 2.

Show your work.

1.





How many counters are there on the left side? 16

How many counters are there on the right side? 16

Why are the amounts of counters on each side the same?

BECAUSE WHAT YOU DO TO THE LEFT MUST = THE RIGHT SIDE SO THE AMOUNTS ARE BALANCED.

What do you think THE PRESERVATION OF EQUALITY means?

TO KEEP THINGS EQUAL ON BOTH SIDES. PRESERVE (KEEP) THE EQUALITY (SAMENESS) OF BOTH SIDES IN AN EQUATION.

Be prepared to discuss your answers ☺.

Consider the following equation:

EX: $4=2+2$

$12 = 3t$ where t is an unknown amount.
To show that equality is preserved, we can add, subtract, multiply or divide the amounts on each side.

Example: if you **add** 3 to each side,
$12 = 3t$ becomes
$12 + 3 = 3t + 3$ Preservation of equality means 3 is added to both sides.

Example: if you **subtract** 2 from each side,
$12 = 3t$ becomes
$12 - 2 = 3t - 2$ Preservation of equality means 2 is subtracted from both sides.

Example: if you multiply by 4
$12 = 3t$ becomes
$12 \times 4 = 3t \times 4$ Preservation of equality means you multiply both sides by 4.
You could also write $12 + 12 + 12 + 12 = 3t + 3t + 3t + 3t$

Example: if you divide by 3
$12 = 3t$ becomes
$\frac{12}{3} = \frac{3t}{3}$ Preservation of equality means you divide both sides by 3.

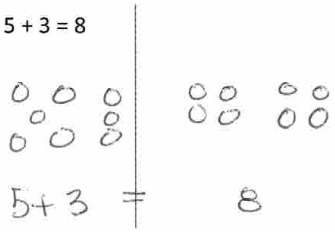
For each example below, the original equation is given with a mathematical change given. Which of the examples preserve equality?

Original	Change	Preserve?	Why?
$x + 4 = 6$	$x + 4 - 2 = 6 - 2$	YES	BECAUSE -2 on both sides of =
$x - 5 = 7$	$x - 5 + 5 = 7 - 5$	NO.	BECAUSE +5 on 1 SIDE + -5 on The oth
$3t = 18$	$4 \times 3t = 4 \times 18$	YES	BECAUSE 4x on both sides of
$12B = 36$	$\frac{12B}{3} = \frac{36}{4}$	NO	BECAUSE -3 on left but -4 on right.

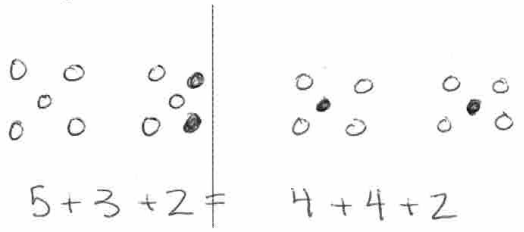
For the equation below, model the equation with counters. Use the counters to show the preservation of equality. Then draw a diagram to record your work by using symbols.

d) Addition

5 + 3 = 8

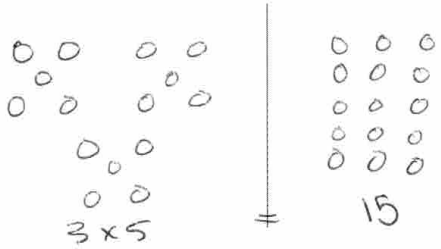


Adding 2

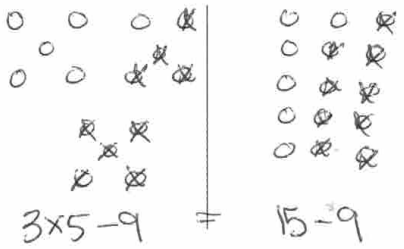


e) Subtraction

3 x 5 = 15

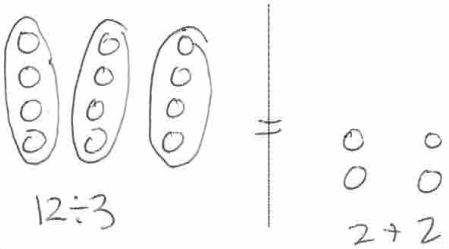


Subtract 9

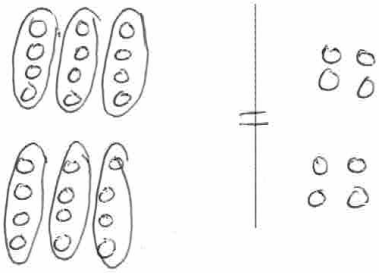


f) Multiplication

12 ÷ 3 = 2 + 2



Multiply x 2.



DOUBLE LEFT SIDE = DOUBLE RIGHT SIDE

24 IN 6 EQUAL GROUPS = 4 + 4

GAME: EQUAL VALUES RECORDING SHEET

Adapted from Page A-43 of "Well Played: Building Mathematical Thinking Through Number Games and Puzzles Grades 3-5"

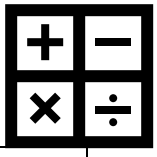
NAMES: _____ DATE: _____

Card #1	Card #2	How do you know the cards make a pack?

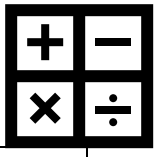
GAME: EQUAL VALUES CARDS

Adapted from Page A-43 of "Well Played: Building Mathematical Thinking Through Number Games and Puzzles Grades 3-5"

540 x 6	500 x 6 + 40 x 6
699 x 4	700 x 4 - 4
17 x 5 x 2	17 x 10
2 x 18 + 2 x 18	18 x 4
98 x 5	100 x 5 – 2 x 5
20 x 6 x 5	6 x 100
23 x 4	20 x 4 + 3 x 4
17 x 5 x 2	17 x 10
34 x 6	6 x 34



23 x 4	20 x 4 + 3 x 4
20 x 7 x 5	7 x 100
50 x 18 x 2	100 x 18
578 x 5	5 x 578
40 x 9 x 5	200 x 9
395 x 5	400 x 5 – 5 x 5
2 x 250 x 5	250 x 10
2 x 7 + 3 x 7	5 x 7
9 x 8	8 x 9
9 x 6	10 x 6 - 6



4 x 3 + 4 x 3	8 x 3
6 x 2 x 5	6 x 10
8 x 7	5 x 7 + 3 x 7
3 x 8	8 x 3
=	=